IAI **Academic year 2022-2023**

FINAL EXAM OF ANALYSIS

Level : L1S2 *Duration : 02 hours*

Exercise 1 : (3mks) True or false (justify your answer).

1- If a function f is strictly increasing and positive on $[0,+\infty[$, then $\lim_{x\to+\infty} f(x) = +\infty$.

2- If a function f has a limit 0 at $+\infty$, then, provided x is large enough, all the real numbers $f(x)$ have the same sign.

3- If a function f has a limit -1 at $+\infty$, then, provided x is large enough, all the real numbers $f(x)$ have the same sign.

Exercise 2 : (10mks) Let f be the real variable function defined as :

$$f(x) = \frac{x^2 - x - 2}{x^2 - x + 1}$$

Let (C) be its representative curve in an orthonormal frame.

1- Find the domain of definition of f.

2- What is the differentiablity set of f ?

3- Find the derivative f' of f. Determine its sign, then deduce the direction of variation of f.

4- Determine the limits of f at the limits of its domain of deefinition.

5- Determine the possible asymptotes and their relative positions with respect to (C).

6- Determine an equation of the tangent T to (C) at $x_0 = 2$.

7- Give the position of (C) relatively to T.

8- Draw on the same drawing, the curve (C), the tangent T and any asymptotes.

9- Could there be an axis of symmetry ? Which ? What is it really ?

Exercise 3 : (2mks) Is the parity of the function

$$f(x) = ln\left(\frac{1+x}{1-x}\right)^{\frac{1}{2}}$$

1- Odd ?

2- Even ?

3- Neither odd nor even ?

Exercise 4 : (5mks) Let f be the function defined on $\mathbb{R} - \{1\}$ as :

$$f(x) = |x - 3| - \frac{2}{x - 1}$$

1- Study the continuity of f at $x_0 = 3$ then on $\mathbb{R} - \{1\}$.

2- Prove that the equation $f(x) = 0$ admits a unique solution in the interval $[3, +\infty[$.

3- Let g be the restriction of f to the interval $[3, +\infty[$. Show that g has an inverse function g^{-1}.

Calculate $g^{-1}\left(\frac{3}{2}\right)$